**Numerical Verification**

Verification and validation (V&V) are the most important means to assess the accuracy and reliability of numerical simulations.

Figure : Stages of CFD Modeling and the

Conceptual Model (Continuum)

Computational Model (Discertized)

Natural Phenomenon

(Reality)

Validation

Qualification

Verification

Programming

Simplification

Simulation

In simple words Verification addresses if the equations are solved correctly and Validations -which is not to be discussed- checks if the equations are proper representative of the real world physics. Lax Equivalence Theorem confirms the consistency and stability of a numerical scheme based on its convergence, as a result the mesh convergence study became a recognized standard in verification of CFD codes. The ratio of consecutive error norms is a perfect vehicle to catch any coding error/algorithm problem. The points need to be considered in mesh any convergence study:

* The FDM and FVM descritzations developed under the assumption of smooth function, the discontinuities and jagged solutions can locally/globally ruin the convergence rate. Linfinity[[1]](#footnote-1)should be included as an ultimate diagnostic tool for local errors and worst case scenario. L2 is more forgiving norm compare to the first error norm “L1”. We recommend L1 as an appropriate global metric of error.[[2]](#footnote-2)
* Convergence ratio in the very coarse grid oscillates around its main value, as the grid size is refined convergence becomes monotone until the mesh size where the machine precision overtakes the truncation error of the numerical scheme, at this point error norms do not change and convergence is zero. Therefore convergence test in extremely fine and coarse grid sizes could be impacted by other factors and it is not reliable.
* Although the convergence is a reliable alarm of a defect, it should not forget in practice a more accurate solver is the goal. Therefore the superiority of methods should be assessed both on convergence and accuracy. Accuracy metrics are error norms as above however in this case they need to be divided by an appropriate scale from the governing equation.
* All the convergence test could be run by a same driver

1. ,,, where *v*= U num - U exact [↑](#footnote-ref-1)
2. It is proven that kL∞ ≤ L2 ≤ L1 ≤ L∞ where k is a constant and 0<k<1, here norms are assumed to be scaled. [↑](#footnote-ref-2)