**Numerical Verification**

Verification and validation (V&V) are the most important steps to assess the accuracy and reliability of numerical simulations.

Figure : Stages of CFD Modeling and the

Conceptual Model (Continuum)

Computational Model (Discertized)

Natural Phenomenon

(Reality)

Validation

Qualification

Verification

Programming

Simplification

Simulation

In simple words in verification correctness of the solution technique used is evaluated and validation is evaluating whether the model is proper representative of the processes involved in the problem. Lax Equivalence Theorem which indicates that for a solver to be stable and consistent it is required to pass the convergence test, is used to assess the consistency and stability of a numerical schemes based on their convergence. As a result, the mesh convergence study became a recognized as the standard verification method of CFD codes. The ratio of consecutive error norms is a perfect vehicle to catch any coding error/algorithm problem. The points need to be considered in any mesh-convergence study include:

* For the FDM and FVM descritzations developed under the assumption of smooth function the discontinuities and jagged initial or boundary conditions can locally or globally decrease the convergence rate. Linfinity[[1]](#footnote-1)should be included as an ultimate diagnostic tool for local errors and worst case scenario. L2 is more forgiving norm compare to the first error norm “L1”. We recommend L1 as an appropriate global metric of error.[[2]](#footnote-2)
* Convergence ratio in a very coarse grid oscillates around its main value, as the grid size is refined convergence becomes monotone until the mesh size reaches to where the machine precision overtakes the truncation error of the numerical scheme, at this point error norms do not change and convergence rate is zero.
* Although the convergence is a reliable warning of a defect, it should not be forgotten that the main goal in practice is a more accurate solver. Therefore the superiority of methods should be assessed both based on convergence and accuracy. Accuracy metrics are error norms as is discussed above however for evaluating the accuracy they should be normalized by an appropriate scale of the solution.
* DISCUSSION ON THE COMPARISION WITH ANALYTICAL SOLUTION, RICHARDSON EXTRAPOLATION, MMS, (MAYBE PSF) here
* All of the convergence tests such as MMS, Richardson Extrapolation, could be run by a same driver.

1. ,,, where *v*= U num - U exact [↑](#footnote-ref-1)
2. It is proven that kL∞ ≤ L2 ≤ L1 ≤ L∞ where k is a constant and 0<k<1, here norms are assumed to be scaled. [↑](#footnote-ref-2)